International Journal of Engineering Science 48 (2010) 708–717

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Subsonic interfacial (Stoneley) waves in anisotropic multiferroic bimaterials with a viscous interface

Yang-Fan Hu [a], Yu-Lan Liu [a], Biao Wang [b,*]

[a] Department of Applied Mechanics & Engineering, Sun Yat-Sen University, 510275 GZ, China
[b] School of Physics and Engineering, Sun Yat-Sen University, 510275 GZ, China

ARTICLE INFO

Article history:
Received 4 December 2009
Accepted 26 January 2010
Available online 9 March 2010
Communicated by M. Kachanov

Keywords:
Interfacial (Stoneley) waves
Multiferroic
Viscosity

ABSTRACT

The problem of subsonic interfacial (Stoneley) wave propagation in anisotropic multiferroic bimaterials with a viscous interface is treated. A concise analytical method is constructed for deduction of possible subsonic interfacial wave with varying viscosity of the interface. A numerical scheme and several calculations are given based on the method, which demonstrate interesting results. For an interface constructed by a piezoelectric half-space and a piezomagnetic half-space, when assumed to be non-viscous, calculation shows that it does not permit any subsonic interfacial wave. Yet when the same interface is assumed to be viscous, at least one possible subsonic interfacial wave speed appears which varies with the viscosity of the interface. By introducing the relation between viscosity of certain adhesives and temperature, the possibility of control of interfacial wave speeds through accommodating the working temperature is put forward.

1. Introduction

Functionally graded multiferroic composites, commonly fabricated as laminated structure, promise extensive application prospect. By coupling the piezoelectric and magnetostrictive properties of the two phases, multiferroic composites possess magnetoelectric (ME) effect [1]. Although the ME effect exists statically, its practical application generally appears in dynamic conditions. In addition, for all artificial composites, dynamical analysis (e.g. nondestructive detection) helps us learn more about the material's property. On the other hand, the ME effect for multiferroic composites is realized mainly through transfer of mechanical stress through the interface. The condition of the interface is thus crucial for the ME effect.

Yet, wave propagation in multiferroic composites is essentially a very complicated problem. On one hand, most multiferroic composites are made of crystals, which falls into transversely isotropic materials, or generally speaking, anisotropic materials. Eringen [2] provides field theories for the study of anisotropic microcontinuum bodies. On the other hand, the effects of piezoelectricity and magnetostrictivity have specific impact on the properties of acoustic wave propagation [3–5]. On this basis, our focus switches to the dynamic behavior on the interface. In a series of two papers, Barnett and Lothe [6,7] constructed the 'impedance method' for surface (Rayleigh) wave and interface (Stoneley) wave propagation in anisotropic material. The mathematical method used in their work derives from Stroh [8] and a comprehensive review given by Chadwick and Smith [9].

In theoretical studies, idealized assumptions are made to simplify the physical problem. In the Stoneley wave problem, interfaces are often assumed to be perfectly bounded [10]. Yet, this is far from the truth. Let alone possible defects, artificial laminated composites are bonded by adhesives, which should be considered viscous at room temperature (300 K) [11]. The

* Corresponding author. Tel.: +86 2084115692.
E-mail address: wangbiao@mail.sysu.edu.cn (B. Wang).

0020-7225/$ - see front matter
doi:10.1016/j.ijengsci.2010.01.006

viscosity of the interface changes the boundary condition of shear stress on it [12], which as a result changes the physical problem and provides a possibility for appearance of new phenomenon. Wang et al. [13] applied the viscous interface condition to the study of two-dimensional Green's function for multiferroic composites and gave some interesting results. In this paper the subsonic interfacial (Stoneley) wave in anisotropic multiferroic bimaterials with a viscous interface is treated.

The paper is arranged as follow: in Section 2, the linearized basic dynamic equations for multiferroic single crystals are given. In Section 3, general solution of the interfacial (Stoneley) waves in anisotropic multiferroic bimaterials with a viscous interface is deduced. In Section 4, a computational scheme using the solution is provided, together with several important numerical results and the physical interpretation of these results. A brief conclusion is drawn in Section 5.

2. Basic formulations

For linearized anisotropic multiferroic single crystal, the equation of equilibrium for the coupled magneto-electro-elastic field can be expressed as:

$$C_{iJKl}u_{K,li} + f_J = \rho \ddot{u}_j, \tag{2.1}$$

where

$$C_{iJKl}, = \begin{cases} C_{ijkl}, & J,K = 1,2,3, \\ e_{lij}, & J = 1,2,3;\; K = 4, \\ e_{ikl}, & J = 4;\; K = 1,2,3, \\ q_{lij}, & J = 1,2,3;\; K = 5, \\ q_{ikl}, & J = 5;\; K = 1,2,3, \\ -\lambda_{il}, & J = 4;\; K = 5 \text{ or } J = 5;\; K = 4, \\ -\varepsilon_{il}, & J,K = 4, \\ -\mu_{il}, & J,K = 5 \end{cases} \tag{2.2}$$

and

$$u_J = \begin{cases} u_j, & J = 1,2,3, \\ \phi, & J = 4, \\ \varphi, & J = 5, \end{cases} \qquad f_J = \begin{cases} f_j, & J = 1,2,3, \\ -f_e, & J = 4, \\ -f_m, & J = 5. \end{cases} \tag{2.3}$$

C_{ijkl}, ε_{ij} and μ_{ij} are the elastic, dielectric, and magnetic permeability tensors, respectively; e_{ijk}, q_{ijk} and λ_{ij} are the piezoelectric, piezomagnetic, and magnetoelectric coefficients, respectively. u_i, ϕ and φ are the elastic displacement, electric potential, and magnetic potential, respectively; f_i, f_e and f_m are the body force, electric charge, and electric current (or called magnetic charge as compared to the electric charge), respectively. $\ddot{u}_j$ indicates second order derivative of u_j with respect to time. $u_{K,i}$ represents derivative of u_K with respect to the component of position x_i. In Eq. (2.1) and the following deduction, repeated indices mean summation, capital subscripts (e.g. J, K in C_{iJKl}) vary from 1 to 5, and lowercase subscripts (e.g. i, l in C_{iJKl}) vary from 1 to 3. Note that in Eq. (2.1), capital subscripts and lowercase subscripts appear in one equation, this means that when J, K = 4 or 5 the terms with lowercase subscripts (e.g. $\rho\ddot{u}_j$ on the right side of the equation) vanish. When the extended body force f_J is zero, Eq. (2.1) is reduced to

$$C_{iJKl}u_{K,li} = \rho \ddot{u}_j. \tag{2.4}$$

The extended elastic coefficient tensor C_{iJKl} in Eq. (2.2) relates the extended strains to the extended stresses by the constitutive relation

$$\sigma_{iJ} = C_{iJKl}\gamma_{Kl}, \tag{2.5}$$

where the extended stresses and strains are defined by

$$\sigma_{iJ} = \begin{cases} \sigma_{ij}, & J = 1,2,3, \\ D_i, & J = 4, \\ B_i, & J = 5, \end{cases} \qquad \gamma_{Ij} = \begin{cases} \gamma_{ij}, & I = 1,2,3, \\ -E_j, & I = 4, \\ -H_j, & I = 5. \end{cases} \tag{2.6}$$

In Eq. (2.6), σ_{ij}, D_i and B_i are the stress, electric displacement, and magnetic induction (i.e. magnetic flux), respectively; γ_{ij}, E_i and H_i are the strain, electric field and magnetic field, respectively. It is observed that various uncoupled cases (i.e. purely elasticity, piezoelectricity, and piezomagneticity can be reduced from Eqs. (2.1)–(2.4) and (2.5) by setting the appropriate coefficients to zero. It is further noticed that the following symmetry relations hold:

$$\begin{aligned} &C_{ijkl} = C_{jikl} = C_{klij}, \\ &e_{kji} = e_{kij}; \quad q_{kji} = q_{kij}, \\ &\varepsilon_{ij} = \varepsilon_{ji}; \quad \lambda_{ij} = \lambda_{ji}; \quad \mu_{ij} = \mu_{ji}. \end{aligned} \tag{2.7}$$

Finally, the extended strains and displacements are related by the geometric equation

$$\gamma_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \\ E_i = -\phi_{,i}; \quad H_i = -\varphi_{,i}. \tag{2.8}$$

3. General solution of the interfacial (Stoneley) waves

The physical problem considered in this paper is depicted in Fig. 1, where two multiferroic half-spaces are boned together, each with arbitrary anisotropy. The Cartesian coordinates ($\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$) are chosen so that $\mathbf{e}_2$ points towards the upper half-spaces and $\mathbf{e}_1$ points towards the direction of propagation of the Stoneley waves. $\mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2$ is not denoted in Fig. 1. $\mathbf{x} = [x_1 x_2 x_3]^T$ is the position vector described in the frame ($\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$). Quantities associated with the upper and lower half-spaces will be followed by subscripts (1) and (2), respectively, illustrated in Fig. 1.

Consider propagation of a harmonic interfacial wave, the corresponding boundary conditions on the interface $x_2 = 0$ are [12]

$$\sigma_{2J}^{(1)} = \sigma_{2J}^{(2)}, \quad (J = 1, 2, 3, 4, 5), \tag{3.1a}$$

$$u_2^{(1)} = u_2^{(2)}; \quad u_4^{(1)} = u_4^{(2)}; \quad u_5^{(1)} = u_5^{(2)}; \quad \sigma_{21}^{(2)} = \eta_1\left(\dot{u}_1^{(1)} - \dot{u}_1^{(2)}\right); \quad \sigma_{23}^{(2)} = \eta_3\left(\dot{u}_3^{(1)} - \dot{u}_3^{(2)}\right), \tag{3.1b}$$

where η_1 and η_3 are the viscous coefficients of the interface in the $\mathbf{e}_1$ and $\mathbf{e}_3$ directions. Eq. (3.1a) guarantees the continuity of stress across the interface. Eq. (3.1b) depicts the conditions for displacements on a viscous interface, where we can see that vertical component u_2, the electric potential u_4 and the magnetic potential u_5 are still required to be continuous across the interface; yet the displacement components on the interface, u_1 and u_3, are allowed to take a jump across the interface, the extent of which controlled by the sheer stress on the interface and the viscous coefficients.

The general solution for the Stoneley wave problem is constructed by combining two Stroh waves with the same phase velocity v on the interface, which gives

$$u_J^{(1)}(\mathbf{x}, t) = \sum_{\alpha=1}^{5} A_{J\alpha}^{(1)} E_\alpha^{(1)} \exp\left[ik(x_1 + p_\alpha^{(1)} x_2 - vt)\right]; \quad x_2 \geqslant 0, \tag{3.2a}$$

$$u_J^{(2)}(\mathbf{x}, t) = \sum_{\alpha=1}^{5} A_{J\alpha}^{*(2)} E_\alpha^{*(2)} \exp\left[ik(x_1 + p_\alpha^{*(2)} x_2 - vt)\right]; \quad x_2 \leqslant 0, \tag{3.2b}$$

where $A_{J\alpha}^{(1)}, A_{J\alpha}^{*(2)}$ and $p_\alpha^{(1)}, p_\alpha^{*(2)}$ are obtained by using the Stroh formalism, whose detailed expressions and physical meaning are given in Appendix A. In Eqs. (3.2a) and (3.2b), the wave number k is real and positive, * denotes complex conjugation, E_K are constants determined by the boundary conditions. Substitution of Eqs. (3.2a) and (3.2b), (2.5) and (2.8) into Eq. (3.1a) gives

$$\sum_{\alpha=1}^{5} L_{J\alpha}^{(1)} E_\alpha^{(1)} = -\sum_{\alpha=1}^{5} L_{J\alpha}^{*(2)} E_\alpha^{*(2)}, \tag{3.3}$$

where

$$L_{J\alpha} = -[C_{2JK1} + p_\alpha C_{2JK2}]A_{K\alpha}. \tag{3.4}$$

Eq. (3.4) gives a relation that depends on p_α. Barnett and Lothe [6] have proved in their work that

$$\mathbf{L}_\alpha = -i\mathbf{Z}\mathbf{A}_\alpha, \tag{3.5}$$

where $\mathbf{Z}$ is called the surface impedance tensor, whose details are given in Appendix B. According to Eq. (3.5), Eq. (3.3) can be transformed as

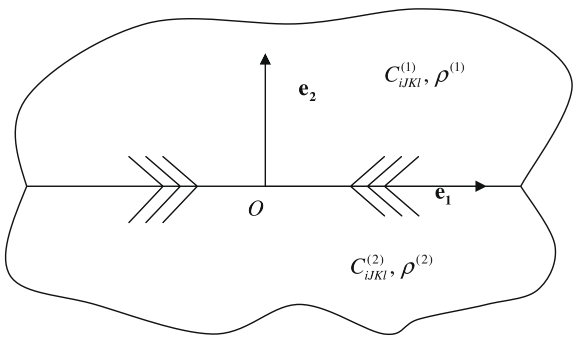

Fig. 1. Geometry associated with the Stoneley wave problem.

$$Z_{JK}^{(1)}U_K^{(1)} = -Z_{JK}^{*(2)}U_K^{*(2)}, \tag{3.6}$$

where

$$U_K^{(1)} = \sum_{\alpha=1}^{5} A_{K\alpha}^{(1)}E_\alpha^{(1)}; \quad U_K^{*(2)} = \sum_{\alpha=1}^{5} A_{K\alpha}^{*(2)}E_\alpha^{*(2)}. \tag{3.7}$$

From (3.1b), we have

$$U_2^{(1)} = U_2^{*(2)}, \quad U_4^{(1)} = U_4^{*(2)}, \quad U_5^{(1)} = U_5^{*(2)}, \quad \eta_1 v\left(U_1^{(1)} - U_1^{*(2)}\right) = Z_{1K}^{(1)}U_K^{(1)}, \quad \eta_1 v\left(U_3^{(1)} - U_3^{*(2)}\right) = Z_{3K}^{(1)}U_K^{(1)}. \tag{3.8}$$

Eq. (3.8) can be recasted into a matrix form as

$$\mathbf{U}^{*(2)} = (\mathbf{I} - \mathbf{D})\mathbf{U}^{(1)}, \tag{3.9}$$

where

$$\mathbf{U} = [U_1 \quad U_2 \quad U_3 \quad U_4 \quad U_5]^T, \tag{3.10}$$

$$\mathbf{D} = \frac{1}{\eta_1 v}\begin{bmatrix} Z_{11}^{(1)} & Z_{12}^{(1)} & Z_{13}^{(1)} & Z_{14}^{(1)} & Z_{15}^{(1)} \\ 0 & 0 & 0 & 0 & 0 \\ \frac{\eta_1}{\eta_3}Z_{31}^{(1)} & \frac{\eta_1}{\eta_3}Z_{32}^{(1)} & \frac{\eta_1}{\eta_3}Z_{33}^{(1)} & \frac{\eta_1}{\eta_3}Z_{34}^{(1)} & \frac{\eta_1}{\eta_3}Z_{35}^{(1)} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{3.11}$$

$\mathbf{I}$ is the fifth-order unit matrix. Substitution of Eq. (3.9) into Eq. (3.6) gives

$$\boldsymbol{\Psi}\mathbf{U}^{(1)} = 0, \tag{3.12}$$

where

$$\boldsymbol{\Psi} = \mathbf{Z}^{(1)} + \mathbf{Z}^{*(2)}(\mathbf{I} - \mathbf{D}). \tag{3.13}$$

Here we name $\boldsymbol{\Psi}$ as the viscous impedance tensor for interfaces. As viscosity does not exist for a free surface, $\boldsymbol{\Psi}$ can be directly called *viscous impedance tensor*. From Eq. (3.12), it is obvious that a subsonic wave solution exists if and only if there exists a phase velocity v_s, such that Eq. (3.12) is satisfied. Yet, From the expression given by Eq. (3.13), it is noticed that for most cases, $\boldsymbol{\Psi}$ is not hermitian, which gives

$$\boldsymbol{\Psi} = \boldsymbol{\Psi}^R + i\boldsymbol{\Psi}^I, \tag{3.14}$$

where both $\boldsymbol{\Psi}^R$ and $\boldsymbol{\Psi}^I$ are both real matrices. Generally speaking, for Eq. (3.12) to have solution, $\mathbf{U}^{(1)} = \mathbf{U}^{(1)R} + i\mathbf{U}^{(1)I}$ must be a complex vector, which requires that E_K and E_K^* given by Eqs. (3.2a) and (3.2b) be complex numbers. For a complex matrix $\boldsymbol{\Psi}$, Eq. (3.12) can be transformed as

$$(\boldsymbol{\Psi}^R + i\boldsymbol{\Psi}^I)(\mathbf{U}^{(1)R} + i\mathbf{U}^{(1)I}) = 0, \tag{3.15}$$

which in term gives

$$\begin{cases} \boldsymbol{\Psi}^R\mathbf{U}^{(1)R} - \boldsymbol{\Psi}^I\mathbf{U}^{(1)I} = 0, \\ \boldsymbol{\Psi}^R\mathbf{U}^{(1)I} + \boldsymbol{\Psi}^I\mathbf{U}^{(1)R} = 0. \end{cases} \tag{3.16}$$

If $\det \boldsymbol{\Psi}^I = \det \boldsymbol{\Psi}^R = 0$ at one phase velocity ($v = v_s$), then v_s enables a interfacial wave. Otherwise, we have from Eq. (3.16)

$$\det[\boldsymbol{\Psi}^I(\boldsymbol{\Psi}^R)^{-1}\boldsymbol{\Psi}^I + \boldsymbol{\Psi}^R] = 0, \quad (v = v_s), \tag{3.17}$$

where $\mathbf{U}^{(1)I}$ belongs to the null-space of $[\boldsymbol{\Psi}^I(\boldsymbol{\Psi}^R)^{-1}\boldsymbol{\Psi}^I + \boldsymbol{\Psi}^R]|v_s$ and $\mathbf{U}^{(1)R} = (\boldsymbol{\Psi}^R)^{-1}\boldsymbol{\Psi}^I\mathbf{U}^{(1)I}$. Eq. (3.17) is deduced for general anisotropic materials. Yet, we know that presently most piezoelectric or piezomagnetic materials are transversely isotropic. For this special kind of material, we find in our calculation that although $\boldsymbol{\Psi}$ in Eq. (3.14) is a complex matrix and also not hermitian, $\det \boldsymbol{\Psi}$ is always real. We present here a brief proof of this conclusion. For a bimaterial made up of a magnetostrictive upper half-space and a piezoelectric lower half-space (both materials are transversely isotropic), $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{*(2)}$ in Eq. (3.13) are of the structure

$$\mathbf{Z}^{(1)} = \begin{bmatrix} R_{11} & I_{12} & I_{13} & 0 & I_{15} \\ -I_{12} & R_{22} & R_{23} & 0 & R_{25} \\ -I_{13} & R_{23} & R_{33} & 0 & R_{35} \\ 0 & 0 & 0 & R_{44} & 0 \\ -I_{15} & R_{25} & R_{35} & 0 & R_{55} \end{bmatrix}; \quad \mathbf{Z}^{*(2)} = \begin{bmatrix} R'_{11} & I'_{12} & I'_{13} & I'_{14} & 0 \\ -I'_{12} & R'_{22} & R'_{23} & R'_{24} & 0 \\ -I'_{13} & R'_{23} & R'_{33} & R'_{34} & 0 \\ -I'_{14} & R'_{24} & R'_{34} & R'_{44} & 0 \\ 0 & 0 & 0 & 0 & R'_{55} \end{bmatrix}, \tag{3.18}$$

where R_{ij}, R'_{ij} $(i,j = 1,2,3,4,5)$ denotes a real number and I_{ij}, I'_{ij} $(i,j = 1,2,3,4,5)$ denotes a pure imaginary number. The structures of the two matrices in Eq. (3.18) are obtained by specifying the expressions of Eqs. (B.1)–(B.4) and (B.5) in Appendix B. Yet, the detailed expressions of the numbers presented in the matrices in Eq. (3.18) are too complicated to be deduced here. Eq. (3.13) can be recated into

$$\boldsymbol{\Psi} = (\mathbf{Z}^{(1)} + \mathbf{Z}^{*(2)}) - \mathbf{Z}^{*(2)}\mathbf{D}, \tag{3.19}$$

where the first terms in the bracket is a hermitian and the second term is given by

$$\mathbf{Z}^{*(2)}\mathbf{D} = \begin{bmatrix} R_{11}R'_{11} - I_{13}I'_{13} & R'_{11}I_{12} + R_{23}I'_{13} & R'_{11}I_{13} + R_{33}I'_{13} & 0 & R'_{11}I_{15} + R_{35}I'_{13} \\ -R_{11}I'_{12} - R'_{23}I_{13} & -I_{12}I'_{12} + R_{23}R'_{23} & -I_{13}I'_{12} + R_{33}R'_{23} & 0 & -I_{15}I'_{12} + R_{35}R'_{23} \\ -R_{11}I'_{13} - R'_{33}I_{13} & -I_{12}I'_{13} + R_{23}R'_{33} & -I_{13}I'_{13} + R_{33}R'_{33} & 0 & -I_{15}I'_{13} + R_{35}R'_{33} \\ -R_{11}I'_{14} - R'_{34}I_{13} & -I_{12}I'_{14} + R_{23}R'_{34} & -I_{13}I'_{14} + R_{33}R'_{34} & 0 & -I_{15}I'_{14} + R_{35}R'_{34} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{3.20}$$

The structure of the 5 * 5 matrix $\boldsymbol{\Psi}$ turns out to be

$$\boldsymbol{\Psi} = \begin{bmatrix} R''_{11} & I''_{12} & I''_{13} & I''_{14} & I''_{15} \\ -I''_{21} & R''_{22} & R''_{23} & R''_{24} & R''_{25} \\ -I''_{31} & R''_{32} & R''_{33} & R''_{34} & R_{35} \\ -I''_{41} & R''_{42} & R''_{43} & R''_{44} & R''_{45} \\ -I''_{51} & R''_{52} & R''_{53} & R''_{54} & R''_{55} \end{bmatrix} \tag{3.21}$$

from which it is easy to prove that $\det \boldsymbol{\Psi}$ is real. In this case, we can use

$$\det \boldsymbol{\Psi} = 0, \quad (v = v_s) \tag{3.22}$$

as the determining equation for v_s.

Eqs. (3.14)–(3.21) and (3.22) provide equations on the possible phase velocity of the interfacial (Stoneley) wave as a function of the viscous coefficients η_1 and η_3. Moreover, in physical chemistry, we know that the viscous coefficients of the adhesives which bonds the two half-space together at the interface are intimately affected by several physical conditions of the surroundings, especially temperature. On the contrary, if the relation of viscosity with temperature $\eta = \eta(T)$ is known, we are able to deduce the variation of phase velocity of the interfacial (Stoneley) wave caused by changes in temperature.

4. Computation

4.1. A computational scheme

The method of viscous impedance tensor provides a concise yet rigorous way for searching Stoneley wave modes in functionally graded multiferroic bimaterials with a viscous interface. Moreover, it avails discussion on the effects of different physical conditions (e..g. viscosity, temperature) on the speed of propagation. A simple scheme is given for this purpose.

1. For the two given half-spaces and given direction of propagation, calculate the 'limiting speed' mentioned in Appendix B. Assume here that the limiting speeds for the upper and lower half-space are $\hat{v}_1$ and $\hat{v}_2$, respectively. One may follow the procedure introduced in Barnett and Lothe [6] or Chadwick and Smith [9].
2. Find the smaller of the two limiting speed, here we assume that $\hat{v}_1 < \hat{v}_2$. For given values of η_1, η_3, in the range of $0 < v \leqslant \hat{v}_1$, calculate the permitted Stoneley wave speed v_s which makes $\det \boldsymbol{\Psi} = 0$. If in the range of $0 < v \leqslant \hat{v}_1$, the value $\det \boldsymbol{\Psi}$ does not change sign, then there is no Stoneley wave for the given η_1, η_3.
3. Change η_1, η_3 and repeat process 1, 2; find the relation between v_s and η_1, η_3.

4.2. Numerical results

We consider the most commonly used functionally graded multiferroic bimaterials made up of piezoelectric $BaTiO_3$ and magnetostrictive $CoFe_2O_4$. The material coefficients are given in Table 1 [14]. As illustrated in Fig. 1, the materials are polarized in the axis $\mathbf{e_2}$.

The interface of this bimaterial made up of $BaTiO_3$ and $CoFe_2O_4$, if without viscosity, does not permit a subsonic Stoneley wave. In the first numerical example, we calculate the effect of viscosity on the speed of Stoneley wave in a rather wide scale. The results are given in Fig. 2.

In reality, the viscosity of any adhesive becomes extremely sensitive to temperature when it cools down. Therefore, in the second calculation, we give another computation on the effect of viscosity on the speed of Stoneley wave in a rather small scale (10–100 P). The results are given in Fig. 3. In the third calculation, influence of viscosity on the wave speed of an interface that initially permits a Stoneley wave is treated. In this numerical example, we consider an infinite $BaTiO_3$ matrix cutted

Table 1
Material coefficients(C_{ij} in 10^9 N/m^2, e_{ij} in C/m^2, q_{ij}in N/Am, ε_{ij} in 10^{-9} C^2/(N m^2), and μ_{ij} in 10^{-6} N s^2/C^2).

	C_{11}	C_{33}	C_{13}	C_{44}	C_{66}	
$BaTiO_3$	166	162	78	43	44.5	
$CoFe_2O_4$	286	269.5	170.5	45.3	56.5	
	e_{31}	e_{33}	e_{15}	q_{31}	q_{33}	q_{15}
$BaTiO_3$	−4.4	18.6	11.6	0	0	0
$CoFe_2O_4$	0	0	0	580.3	699.7	550
	ε_{11}	ε_{33}	μ_{11}	μ_{33}		
$BaTiO_3$	11.2	12.6	5	10		
$CoFe_2O_4$	0.08	0.093	−590	157		

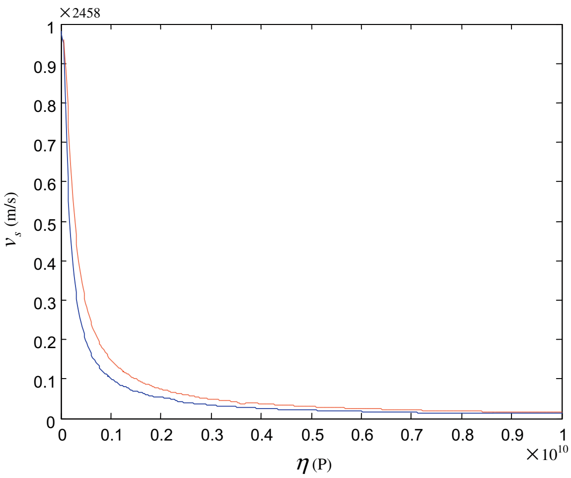

Fig. 2. Variation of the phase velocity of interfacial (Stoneley) wave propagating in multiferroic bimaterials made of $BaTiO_3$ and $CoFe_2O_4$ with viscosity. Two possible speeds are shown from the picture. The one depicted by broken line appear later (at about 8×10^7 P). Values in y-axis should be multiplied by 2458, which is the 'limiting speed' mentioned in Appendix B.

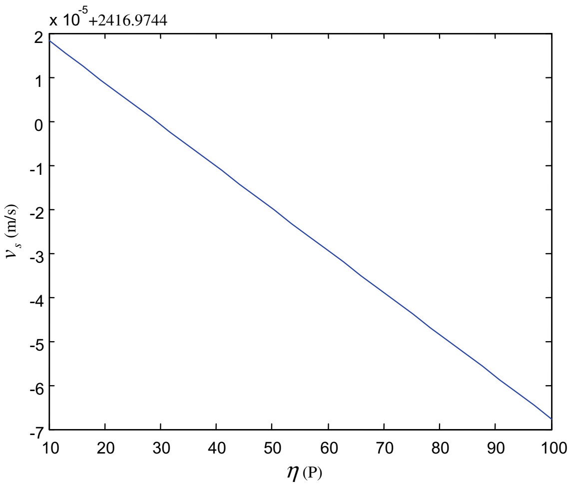

Fig. 3. Variation of the phase velocity of interfacial (Stoneley) wave propagating in multiferroic bimaterials made of $BaTiO_3$ and $CoFe_2O_4$ with viscosity. As the values of viscosity is rather small compared with the ones in Fig. 2, only one possible phase velocity exist. Values in y-axis should be added by 2416.9744.

into two half-spaces at the plane $x_2 = 0$, the material is polarized in axis x_2 as shown in Fig. 1. Fig. 4 shows the calculation results.

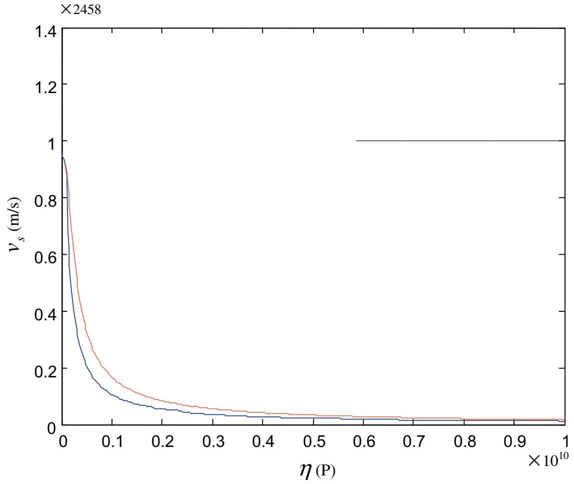

Fig. 4. Variation of the phase velocity of interfacial (Stoneley) wave propagating in an infinite $BaTiO_3$ matrix cutted into two half-spaces at the plane $x_2 = 0$ with viscosity. Two possible speeds exist at the beginning and then another one appear at certain point. Yet, from our results it is observed that the value of the new possible speed is slightly larger then the 'limiting speed' and finally equals to it when viscosity reaches infinity.

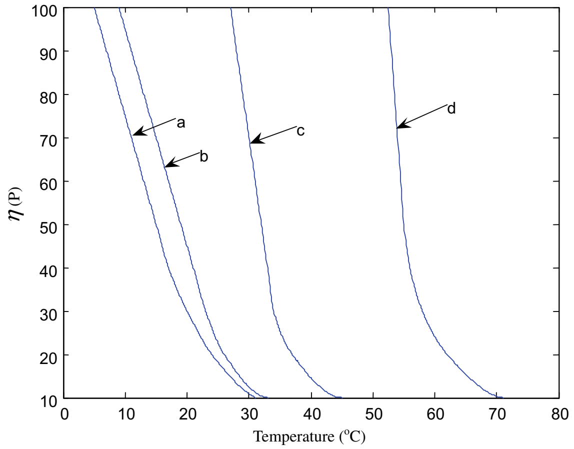

Fig. 5. Variation of liquid epoxy resin viscosity with temperature: (a) liquid diglycidyl ether plus glycidyl ester of a tertiary carboxylic acid as a reactive diluent; (b) liquid diglycidyl ether plus dibutyl phthalate as a plasticizer; and (c) unmodified diglycidyl either; semisolid diglycidyl either.

Fig. 5 shows a rapid decrease of the viscosity of epoxy resin system with temperature [11]. Then, by using this relation between temperature and viscosity of certain adhesives, the variation of phase speed as a function of the temperature is pictured. The results are given in Fig. 6. Our calculation procedure is applicable to interfaces constructed by any "reduced material", including piezoelectric/piezomagnetic materials, elastic materials (with triclinic, monoclinic, rhombic, trigonal, tetragonal, transversely isotropic, cubic and isotropic symmetry).

4.3. Discussion

From Fig. 2, we see that when viscosity is considered, subsonic Stoneley waves emerges for interfaces that originally forbid their propagation. In Fig. 2, two possible Stoneley waves are shown with different phase speed. Note that the one depicted with real line appears in the very beginning (at the first point calculated, the viscosity is assumed to be 10 P), while the other one depicted with broken line appears much later (around 8×10^7 P). Both possible wave speeds decrease with increasing viscosity and finally vanish, so when viscosity reaches infinity (i.e. non-viscosity), no available wave speed exists.

In Fig. 4, we see that one more available wave speed (dashdotted) appear with increasing viscosity. As the viscosity approaches infinity, the new wave speed is the only one that remains, and equals to the wave speed obtained for non-viscous interface Stoneley wave. Yet, from our results it is observed that the value of the new possible speed is slightly larger than the 'limiting speed', which implicates that this velocity transform from supersonic to subsonic speed.

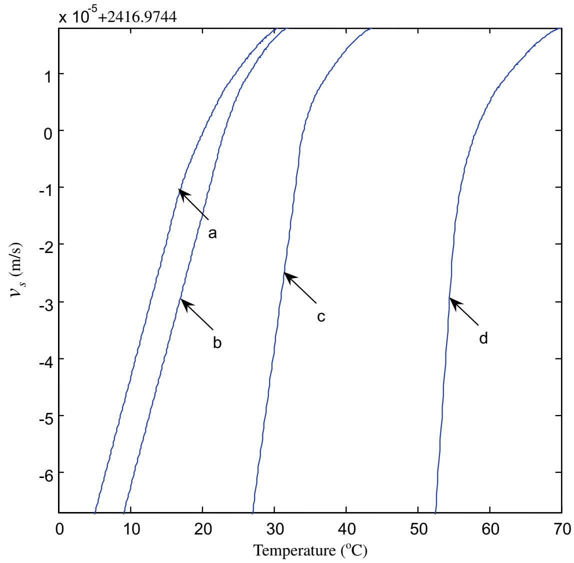

Fig. 6. Variation of the phase velocity of interfacial (Stoneley) wave propagating in multiferroic bimaterials made of $BaTiO_3$ and $CoFe_2O_4$ with temperature. The interface is adhered by different liquid epoxy resins as shown in Fig. 5. Values in y-axis should be added by 2416.9744.

It should be noted that the viscosity of epoxy resin shown in Fig. 5 are tested in uncured state, for cured adhesives the viscosity might present different properties. Yet, this is not a problem covered in this paper.

5. Conclusion

We have introduced the notion of viscous impedance tensor $\boldsymbol{\Psi}$, which permits settling the question of existence of subsonic Stoneley wave in anisotropic multiferroic bimaterials with a viscous interface. Although the complexity of the tensor $\boldsymbol{\Psi}$ prevent further discussion on the problem of uniqueness, it is clear from the numerical results that for nonsingular viscosity, at least one subsonic wave speed is permitted, and in our results, at most there are three possible subsonic wave speeds. An important result is discovered in the first numerical example that for a non-viscous interface that does not permit a Stoneley wave, the appearance of viscosity of the interface enables at least one possible Stoneley wave, and the wave speed varies as a function of viscosity of the interface. In the last computation, we show a picture demonstrating the effect of change in working temperature on the Stoneley wave speed propagating in a viscous interface.

Acknowledgement

The work was supported by the NSFC (National Natural Science Foundation of China) through the funds (10732100, 10831160504, and 10972239).

Appendix A. Stroh formalism

Consider an extended displacement field described by

$$u_J = A_J f(x_1 + px_2 - vt), \tag{A.1}$$

where p is an unknown to be decided. (For a more generalized description, please refer to [8,15].) Substitution of Eq. (A.1) into Eq. (2.1) gives

$$[C_{1JK1}] + ([C_{1JK2}] + [C_{2JK1}])p + [C_{2JK2}]p^2 - \rho v^2 \mathbf{E} = 0, \tag{A.2}$$

where

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{A.3}$$

and $[C_{1JK1}]$ demotes a second order matrix with C_{1JK1} as its element in the Jth line and Kth column. For a given phase velocity v, Eq. (A.2) provides a eigenvalue equation which determines p. For subsonic problem, p appear as five pairs of conjugates. Therefore, Eq. (A.1) offers a class of functions that, if its parameter p properly chosen, intrinsically satisfies the equilibrium equation. Consider a wavelike solution $f(\omega) = \exp(ik\omega)$, where k is real and positive, Eq. (A.2) can be recasted into a standard eigenvalue problem by using the Stroh's formalism.

$$\mathbf{N}\xi = p\xi, \tag{A.4}$$

where

$$\mathbf{N} = -\begin{bmatrix} (nn)^{-1}(nm) & (nn)^{-1} \\ (mn)(nn)^{-1}(nm) - (mm) + \rho v^2\mathbf{E} & (mn)(nn)^{-1} \end{bmatrix} \tag{A.5}$$

is a 10 * 10 matrix and the symbol (nm) denotes a 5 * 5 matrix whose components are given by

$$(nm)_{JK} = n_i C_{iJKl} m_l. \tag{A.6}$$

In Eq. (A.4)

$$\xi = \begin{bmatrix} \mathbf{A} \\ \mathbf{L} \end{bmatrix}, \tag{A.7}$$

where $\mathbf{L}$ is a vector whose components are given by

$$L_J = -n_i C_{iJKl}(m_l + pn_l)A_K. \tag{A.8}$$

Eq. (A.4) determines five pairs of eigenvalues p and its corresponding eigenvector ξ. Note that functions with the form Eq. (A.1) can be determined merely by substitution into Eq. (2.1), which has nothing to do with the boundary conditions. Therefore, since the equations of equilibrium did not change in the problem of subsonic interfacial (Stoneley) wave propagating in anisotropic multiferroic bimaterials with a viscous interface, in Eqs. (3.2a) and (3.2b) p_α and $A_{J\alpha}$ are obtained by the same procedure introduced in Appendix A.

Appendix B. Surface impedance tensor

Barnett and Lothe [6,7] gives the surface impedance method for calculation on the surface wave propagation in anisotropic elastic half-space. Space limitation precludes reproduction of their work, but the main procedure for solving the surface impedance tensor $\mathbf{Z}$ will be presented in this section, along with several notes on the validity of this procedure.

$$\mathbf{Z} = -(\mathbf{Q}^{-1} + i\mathbf{Q}^{-1}\mathbf{S}), \tag{B.1}$$

where

$$\mathbf{Q} = -\frac{1}{\pi}\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} [\![\mathbf{ss}]\!]^{-1} d\phi, \tag{B.2}$$

$$\mathbf{S} = -\frac{1}{\pi}\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} [\![\mathbf{ss}]\!]^{-1}[\![\mathbf{sr}]\!] d\phi \tag{B.3}$$

and the operator $[\![\mathbf{ab}]\!]$ is defined as a matrix, whose components are

$$[\![\mathbf{ab}]\!]_{JK} = a_i(C_{iJKl} - \rho v^2 \delta_{i1}\delta_{l1}\delta_{jk})b_l. \tag{B.4}$$

In Eq. (B.3), $(\mathbf{s}, \mathbf{r})$ form an orthogonal pair of real unit vectors in the $\mathbf{e_1} - \mathbf{e_2}$ plane such that

$$\mathbf{s} = -\mathbf{e_1}\sin\phi + \mathbf{e_2}\cos\phi; \quad \mathbf{r} = \mathbf{e_1}\cos\phi + \mathbf{e_2}\sin\phi. \tag{B.5}$$

The surface impedance tensor $\mathbf{Z}$ can be obtained through the process Eq. (B.5) to Eq. (B.1). Several notes should be stated about the validity of this process

1. The $\mathbf{e}_1 - \mathbf{e}_2$ plane is illustrated in Fig. 1, where $\mathbf{e}_1 = [1\,0\,0]^T$; $\mathbf{e}_2 = [0\,1\,0]^T$. Eq. (B.4) is a simplified result constructed in this set of coordinates. For a more generalized expression please refer to Barnett and Lothe [6,7].
2. $[\![\mathbf{ss}]\!]^{-1}$ in Eq. (B.2) denotes inverse matrix of $[\![\mathbf{ss}]\!]$, and it exists as long as $0 \leqslant v \leqslant \hat{v}$, where $\hat{v}$ is the so-called 'limiting speed'. The proof of this corollary was given in detail in Chadwick and Smith [9]. In other words, this procedure for obtaining $\mathbf{Z}$ is valid only if $0 \leqslant v \leqslant \hat{v}$, or we can say if the wave travels under subsonic speed.
3. As long as $0 \leqslant v \leqslant \hat{v}, \mathbf{Z}$ is hermitian because $\mathbf{Q}$ is symmetric and $\mathbf{Q}^{-1}\mathbf{S}$ is antisymmetric.
4. Any conclusion in Barnett and Lothe [6,7] or Chadwick and Smith [9] based on the condition that $[\![\mathbf{ss}]\!]$ is positive definite is no longer valid for multiferroic materials with an extended elastic tensor C_{iJKl}.

References

[1] Y. Benveniste, Magnetoelectric effect in fibrous composites with piezoelectric and piezomagnetic phases, Physical Review B 51 (1995) 16424–16427.
[2] A.C. Eringen, Microcontinuum Field Theories: Foundations and Solids, Springer-Verlag, New York, Inc., 1999.
[3] W.J. Feng, J. Jin, E. Pan, Stoneley (interfacial) waves between two magneto-electro-elastic half planes, Philosophical Magazine 88 (2008) 1801.
[4] A. Melkumyan, On diffraction of acoustic and electric waves in piezoelectric medium by an absorbent half-plane electrode, International Journal of Solids and Structures 44 (2007) 3594.
[5] B.A. Auld, Acoustic Fields and Waves, vols. 1&2, John Wiley, NY, 1973.
[6] D.M. Barnett, J. Lothe, Free surface (Rayleigh) waves in anisotropic elastic half-spaces: the surface impedance method, Proceedings of the Royal Society A 402 (1985) 135–152.
[7] D.M. Barnett, J. Lothe, Considerations of the existence of interfacial (Stoneley) waves in bonded anisotropic elastic half-spaces, Proceedings of the Royal Society A 402 (1985) 153–166.
[8] A.N. Stroh, Steady state problems in anisotropic elasticity, Journal of Mathematical Physics 41 (1962) 77–103.
[9] P. Chadwick, G.D. Smith, Surface waves in anisotropic elastic materials, Advances in Applied Mechanics. 17 (1977) 303–376.
[10] S.K. Tomar, G. Monika, Reflection and transmission of waves from a plane interface between two microstretch solid half-spaces, International Journal of Engineering Science 43 (2005) 139–169.
[11] E.M. Petrie, Epoxy Adhesive Formulations, McGraw-Hill, NY, 2006.
[12] M.F. Ashby, Boundary defects and atomistic aspects of boundary sliding and diffusional creep, Surface Science 31 (1972) 498–542.
[13] X. Wang, E. Pan, J.D. Albrecht, Two-dimensional Green's functions in anisotropic multiferroic bimaterials with a viscous interface, Journal of Mechanics and Physics of Solids 56 (2008) 2863–2875.
[14] E. Pan, Exact solution for simply supported and multilayered magneto-electro-elastic plates, Journal of Applied Mechanics 68 (2001) 608–618.
[15] D.M. Barnett, Bulk, surface, and interfacial waves in anisotropic linear elastic solids, International Journal of Solids and Structures 37 (1-2) (2000) 45–54.